

Midterm exam for ISU Economics 671, Econometrics 1

General points about the exam:

- Please answer on a separate sheet (or sheets) of paper.
 - This exam has 4 questions worth 5 points each.
 - You may leave if you finish the exam early.
 - You will not receive full credit for any answer unless you explain it, even if your calculations are correct.
 - Answers that can not be correct (negative variances, negative pdfs, etc.) will be graded especially critically if you do not acknowledge that the answer is impossible.
- 1) Let $Y_i = \beta x_i + u_i$ for $i = 1, \dots, n$ with $x_i > 0$ for all i , and suppose that $u_i = \rho u_{i-1} + \varepsilon_i$ with the $\varepsilon_i \sim i.i.d. N(0, \sigma^2)$. If $\hat{\beta}$ is the OLS estimator of β , prove that the variance of $\hat{\beta}$ is larger than usual if $\rho > 0$.
 - 2) Suppose that we have three random variables Y , X_1 , and X_2 such that

$$E(Y | X_1, X_2) = \beta_0 + \beta_1 X_1 + \beta_2 X_2$$

with β_1 and β_2 nonzero and X_1 and X_2 independent. Then we can regress Y on X_1 to get an unbiased estimator of β_1 , and the Gauss-Markov theorem states that this is the BLUE. But we can regress Y on X_1 and X_2 to get an estimator for β_1 , and the Gauss-Markov theorem states that this estimator is the BLUE as well. What's going on?

- 3) Suppose that X_1, \dots, X_n and Y_1, \dots, Y_n are independent sequences of random variables and $X_i \sim i.i.d. N(\mu_1, \sigma^2)$ and $Y_i \sim i.i.d. N(\mu_2, \sigma^2)$, but that we also know that $\mu_1 \geq \mu_2$. Please derive the Maximum Likelihood Estimators of μ_1 , μ_2 , and σ^2 .
- 4) State and prove the Law of Iterated Expectations.