

Midterm exam for ISU Economics 671, Econometrics 1 (Tuesday, October 18, 9:00–10:50 a.m.)

- This exam has four questions. Please check that you are not missing any.
- You may leave if you finish the exam early.
- The exam will have a total of 25 points. The first three questions are worth 5 points each, and the last question is worth 10 points.
- You will not receive full credit for any answer unless you explain it, even if your calculations are correct.
- Answers that can not be correct (negative variances, negative pdfs, etc.) will be graded especially critically if you do not acknowledge that the answer is impossible.

- 1) Suppose that X_1 and X_2 are independent univariate Normal random variables. Prove that $X_1 + X_2$ is Normal and find its mean and variance.

2) State and prove Chebychev's inequality.

- 3) Let $X = (X_1, \dots, X_n)$ be a random sample and let θ be some parameter of interest. For each θ_0 , let $A(\theta_0)$ be the acceptance region of a level α test of the null hypothesis that $\theta = \theta_0$. For each sample x , define a set $C(x)$ in the parameter space by

$$C(x) = \{\theta_0 : x \in A(\theta_0)\}.$$

Prove that the random set $C(X)$ is a $1 - \alpha$ confidence set for the parameter θ .

- 4) Suppose that X_1, \dots, X_n are i.i.d. $\text{uniform}(a, b)$.
- a) Please derive the MLE of $E X_i$.
 - b) Please derive the LRT of the null hypothesis $E X_i = 0$ against the two-sided alternative $E X_i \neq 0$. If you can calculate the critical value for a test at the 90% confidence level, do so. But if you can't, just present the statistic.