Final exam for ISU Economics 671, Econometrics 1 (December 16, 2011)

- This exam has one page(s) and three questions. Please check that you are not missing anything.
- You have until 9:30 a.m. to complete the exam and may leave if you finish before that.
- The exam will have a total of 25 points. Questions 1 and 2 are worth ten points each and 3 is worth five points.
- You will not receive full credit for any answer unless you explain it, even if your calculations are correct.
- Answers that can not be correct (negative variances, negative pdfs, etc.) will be graded especially critically if you do not acknowledge that the answer is impossible.
- Please write as well and as neatly as you can: try to make your lines horizontal, your words coherent, and your variables legible.

1) Suppose that you are interested in the model

$$y_i = \beta_0 + x_i\beta_1 + \varepsilon_i$$

where x_i is a random scalar, $\mathrm{E}(\varepsilon_i \mid X) = 0$ and the observations are i.i.d. But suppose that we do not observe x_i directly but instead we observe $w_i = x_i + u_i$, where u_i is another error term.

a) Prove that, if u_i is perfectly correlated with x_i, OLS is essentially consistent in the following sense: let $\dot{x}_i = (x_i - \mathrm{E}\, x_i)/\operatorname{sd}(x_i)$ and let $\dot{w}_i = (w_i - \mathrm{E}\, w_i)/\operatorname{sd}(w_i)$. Then the OLS coefficient in the regression of y_t on $\dot{x}_i$ is the same as that of the regression of y_i on $\dot{w}_i$.

b) Prove that, if u_i is independent of x_i and ε_i, OLS is inconsistent even after standardizing the variables.

c) Derive the MLE of β_1 under the assumption that ε_i and u_i are independent mean-zero normal and determine whether it is consistent.

d) Suppose now that ε_i and u_i are independent, mean zero normal random variables, but now assume that we have another regressor $z_i = x_i + v_i$, where v_i is another mean-zero, normal error term that's independent of ε_i and u_i. Derive the MLE of β_1 and determine whether it is consistent. For bonus points and well-deserved pride: is it asymptotically normal?

2) Suppose that we have the model

$$y_i = \beta_0 + \beta_1 x_{1i} + \beta_2 x_{2i} + \beta_3 x_{3i} + \varepsilon_i$$

where $\mathrm{E}(\varepsilon_i \mid X) = 0$ but the errors may be heteroskedastic.

a) Suppose you want to test the null hypothesis $\beta_i \leq 0$ for all $i = 1, \dots, 3$ against the alternative that $\beta_i > 0$ for at least one i (note that this is a single null hypothesis against a single alternative). How could you use the multiple hypothesis techniques we discussed in class to test this hypothesis?

b) If this procedure rejects the null that all of the β_i are weakly negative, do we know why? Specifically, do we know which of the β_i is positive? In what sense?

c) Please write, in as much detail as you can, R pseudo-code to implement your answer to the first part of the question.

3) State and prove the Gauss-Markov theorem.