Final exam for ISU Economics 671, Econometrics 1 (December 18, 2009)

- This exam has four pages and four questions. Please check that you are not missing anything.
- You have until 9:30 to complete the exam and may leave if you finish before that.
- The exam will have a total of 25 points. Questions 1, 3, and 4 are worth five points each and Question 2 is worth ten points.
- You will not receive full credit for any answer unless you explain it, even if your calculations are correct.
- Answers that can not be correct (negative variances, negative pdfs, etc.) will be graded especially critically if you do not acknowledge that the answer is impossible.

1) State and prove the Gauss-Markov theorem.

2) Suppose that you estimate the model

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \varepsilon_i$$

with OLS and calculate the F-test for the null hypothesis $\beta_1 = 1$. The p-value for this test is 0.03 and $\hat{\beta}_1$ is 0.54. In the following questions, you can assume that all of the necessary OLS assumptions hold.

a) How would you use this information to test against the one-sided alternative that $\beta_1 > 1$ at the 5% level? Do you reject the null in favor of this alternative?

b) How does your answer change if the alternative is $\beta_1 < 1$? Do you reject in favor of this alternative?

c) Based on your answers to the previous two questions, please outline a procedure to test the general null hypothesis that $R\beta \leq q$, where the inequality holds element by element. Obviously, there is not enough time for you to work out a complete solution right now, so write down the key steps first.

3) The following question is taken from Greene's textbook: In the December, 1969, *American Economic Review* (pp. 886-896), Nathaniel Leff reports the following least squares regression results for a cross section study of the effect of age composition on savings in 74 countries in 1964:

$$\ln S/Y = 7.3439 + 0.1596 \ln Y/N + 0.0254 \ln G - 1.3520 \ln D_1 - 0.3990 \ln D_2$$
$$\ln S/N = 2.7851 + 1.1486 \ln Y/N + 0.0265 \ln G - 1.3438 \ln D_1 - 0.3966 \ln D_2$$

where S/Y = domestic savings ratio, S/N = per capita savings, Y/N = per capita income, D_1 = percentage of the population under 15, D_2 = percentage of the population over 64, and G = growth rate of per capita income. Are these results correct? Explain.

4) Suppose that you want to estimate the relationship between two variables, Y and X_1, while controling for the effects of p possible other regressors. Please describe how you could use the AIC to determine which variables to include in your model. Also write down the R code for a function that would calculate this model and return its coefficient estimates; please return "0" as the coefficients for variables not included in the model. Please fill in as many details as you can, but don't worry if you can not remember the exact name or syntax of every command that you want to use.

Remember that the AIC formula can be written as

$$\ln(\hat{\varepsilon}'\hat{\varepsilon}/n) + 2k/n$$

where k is the number of regressors in the model.