Midterm exam for ISU Economics 671, Econometrics 1

General points about the exam:

- Please answer on a separate sheet (or sheets) of paper.
- This exam has 3 questions worth 5 points each.
- You may leave if you finish the exam early, but see the next point.
- You will not receive full credit for any answer unless you explain it, even if your calculations are correct.
- Answers that can not be correct (negative variances, negative pdfs, etc.) will be graded especially critically if you do not acknowledge that the answer is impossible.

1) Suppose that we have the linear relationship

$$Y_i = \beta_0 + \beta_1 X_{i1} + \cdots + \beta_{k-1} X_{i,k-1} + \varepsilon_i$$

where the usual OLS assumptions are satisfied (ε_i is i.i.d. $N(0, \sigma^2)$ given X_i, for example). Now consider the following null hypotheses:

i. H_0: $\beta_i \leq 0$ for all $i = 0, \ldots, k-1$ against the alternative that $\beta_i > 0$ for at least one i.

ii. H_0: $\beta_i \leq 0$ for at least one i in $0, \ldots, k-1$ against the alternative that $\beta_i > 0$ for all i.

Note that these hypotheses are different than what we covered in lecture. We're going to look at tests for these hypotheses that are based on the individual t tests—let t_i denote the t-test statistic for the null $\beta_i \leq 0$ against the one-sided alternative.

a) Please show that, if each t_i is individually valid, then the rule *reject the joint null if any t_i rejects at level α/k, accept it if all of the individual t_i's accept*, is a valid level-α test for the null hypothesis in i.

b) Please show that, if each t_i is individually valid, then the rule *reject the joint null if every t_i rejects at level α, accept it if any of the t_i's fails to reject*, is a valid level-α test for the null hypothesis in ii.

The first procedure is called the *Union-Intersection Test* and the second is the *Intersection-Union Test*.

2) Assume that sufficient conditions hold to ensure that

$$\sqrt{n}(\hat{\beta} - \beta) \to^d N(0, \Sigma)$$

where $\hat{\beta}$ is an OLS estimator. Please explain how to use the δ-method to test the nonlinear hypothesis $g(\beta) = c$, where g is continuously differentiable.

3) This question is indirectly motivated by Monday's seminar, which conducted a time-series analysis. Suppose that we observe $X_1, \ldots, X_n \sim i.i.d.\ (\mu, \sigma^2)$, but instead of studying X_t, we want to study the change in X_t, $\Delta X_t \equiv X_t - X_{t-1}$. Let $Z_t = \Delta X_t$ for $t = 2, \ldots, n$.

a) What is the probability limit of $\bar{Z} = \frac{1}{n-1} \sum_{t=2}^{n} Z_t$?

b) Is $\sqrt{n}(\bar{Z} - \operatorname{E} \bar{Z})$ asymptotically normal (i.e. does it obey a CLT)? Why or why not?