Midterm exam for ISU Economics 671, Econometrics 1 (Tuesday, October 18, 9:00–10:50 a.m.)

General points about the exam:

- Please answer on a separate sheet (or sheets) of paper.
- This exam has four questions. Please check that you are not missing any.
- You may leave if you finish the exam early.
- The exam will have a total of 25 points. Questions 1, 3, and 4 are worth 5 points each, and Question 2 is worth 10 points.
- You will not receive full credit for any answer unless you explain it, even if your calculations are correct.
- Answers that can not be correct (negative variances, negative pdfs, etc.) will be graded especially critically if you do not acknowledge that the answer is impossible.

If you get stuck on a problem, you may want walk yourself through the following checklist and ask yourself the questions listed at each step (taken from G. Polya, *How to Solve It*, 2nd edition, Princeton University Press, 1957, ISBN 0-691-08097-6).

1) *Understanding the problem.* First you have to understand the problem.
   - What is the unknown? What are the data? What is the condition?
   - Is it possible to satisfy the condition? Is the condition sufficient to determine the unknown? Or is it insufficient? Or redundant? Or contradictory?
   - Draw a figure. Introduce suitable notation.
   - Separate the various parts of the condition. Can you write them down?
2) *Devising a plan.* Second, find the connection between the data and the unknown. You may be obliged to consider auxiliary problems if an immediate connection cannot be found. You should obtain eventually a plan of the solution.
   - Have you seen it before? Or have you seen the same problem in a slightly different form?
   - Do you know a related problem? Do you know a theorem that could be useful?
   - Look at the unknown! And try to think of a familiar problem having the same or a similar unknown.
   - Here is a problem related to yours and solved before. Could you use it? Could you use its result? Could you use its method? Should you introduce some auxiliary element in order to make its use possible?
   - Could you restate the problem? Could you restate it still differently? Go back to definitions.
   - If you cannot solve the proposed problem try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem? Keep only a part of the condition, drop the other part; how far is the unknown then determined, how can it vary? Could you derive something useful from the data? Could you think of other data appropriate to determine the unknown? Could you change the unknown or data, or both if necessary, so that the new unknown and the new data are nearer to each other?
   - Did you use all the data? Did you use the whole condition? Have you taken into account all essential notions involved in the problem?
3) *Carrying out the plan.* Third, carry out your plan of the solution and check each step.
   - Can you see clearly that each step step is correct?
   - Can you prove that it is correct?
4) *Looking back.* Fourth, examine the solution obtained.
   - Can you check the result? Can you check the argument?
   - Can you derive the solution differently? Can you see it at a glance?
   - Can you use the result, or the method, for some other problem?

1) Suppose that X and Y are correlated normal random variables with the same variance. Prove that $X+Y$ and $X-Y$ are independent.
2) Suppose that $X_1, \ldots, X_n \sim$ i.i.d. uniform(a,b)

   a) Show that $(\min_i X_i, \max_i X_i)$ is the maximum likelihood estimator of (a, b).
   b) Prove that the MLE is consistent.
   c) Please find the asymptotic distribution of the MLE. You will need to rescale the estimator to find an asymptotic distribution. Two hints: it is not asymptotically normal; and you may need to use the result $(1 + \frac{1}{n})^n \to e$ as $n \to \infty$.
   d) Use your answer to the previous question to construct a two-sided 90% confidence interval for b.
   e) When we derived the asymptotic distribution earlier, we used only some aspects of the assumption that $X_i \sim$ uniform(a,b). Can you show that $\max_i X_i$ and $\min_i X_i$ have the same asymptotic distribution that we derived above under weaker assumptions?

3) Suppose that $X_1, \ldots, X_n \sim N(\theta, \theta)$. Please find the MLE for θ and prove that it is consistent and asymptotically normal.
4) State and prove the Cauchy-Schwarz inequality.