Midterm exam for ISU Economics 671, Econometrics 1 (October 7, 2009)

- This exam has four pages and four questions. Please check that you are not missing anything.
- You have until 10:50 to complete the exam and may leave if you finish before that.
- Questions 1–3 are each worth five points and question 4 is worth ten points.
- You will not receive full credit for any answer unless you explain it, even if your calculations are correct.
- Answers that can not be correct (negative variances, negative pdfs, etc.) will be graded especially critically if you do not acknowledge that the answer is impossible.

Q1. A couple decides to continue to have children until a daughter is born. What is the expected number of children for this couple? To get a number, it may help to remember the relationship

$$\sum_{j=0}^{k} c^j = \frac{1 - c^{k+1}}{1 - c}.$$

Q2. Suppose that Z is a random variable with mean μ and variance σ and let $\varepsilon_1, \varepsilon_2, \ldots$ be a sequence of i.i.d. random variables with mean zero and variance 1 that are independent of Z. Show that the sequence $X_i = Z + \varepsilon_i$ obeys the following weak law of large numbers: as $n \to \infty$, $n^{-1} \sum_{i=1}^{n} \big[X_i - E(X_i \mid Z)\big] \to 0$ in probability. Note that $X_1, X_2, \ldots$ is **not** an i.i.d. sequence, but is an example of an *exchangeable* sequence of random variables.

Q3. Suppose that X is a vector of n random variables and has mean μ (an n-vector) and variance Σ (an $n \times n$ matrix), and let A be another $n \times n$ matrix. Calculate the mean of the quadratic form $X'AX$.

Q4. Let $X_1, \dots, X_n$ be a random sample from the gamma(α, β) distribution, where α is a known integer and β is unknown. Calculate the maximum likelihood and method of moments estimators of β. Remember, a gamma(α, β) random variable has mean $\alpha\beta$, variance $\alpha\beta^2$, and density

$$f(x) = \frac{1}{\Gamma(\alpha)\beta^\alpha} x^{\alpha-1} e^{-x/\beta}, \quad 0 \le x < \infty, \quad \alpha, \beta > 0.$$

Then use either one of these estimators to construct an approximate one-sided 95% confidence interval for β of the form $(0, \beta_U]$. You can decide which of the two estimators you want to use for this interval.